

Ant Colony Optimization Approach for Optimizing Traffic Signal Timings

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Abstract

In urban networks, traffic signals are used to control vehicle movements so as to reduce congestion, improve safety, and enable specific strategies such as minimizing delays, improving environmental pollution, etc (Teklu et al., 2007). Due to the increasing in the number of cars and developing industry, finding optimal traffic signal parameters has been an important task in order to use the network capacity optimally. Through the last decade, developments in communications and information technologies have improved the classical methods for optimising the traffic signal timings toward the intelligent ones. There is an important interaction between the signal timings and the routes chosen by individual road users in road networks controlled by fixed time signals. The mutual interaction leads to the framework of a leader-follower or Stackelberg game, where the supplier is the leader and the user is the follower (Fisk, 1984). Network design problem (NDP) that it may contain the signal setting problem is characterized by the so called bilevel structure. Bi-level programming problems generally are difficult to solve, because the evaluation of the upper-level objective involves solving the lower level problem for every feasible set of upper level decisions (Sun et al., 2006). On the upper level, a transport planner designs the network. Road users respond to that design in the lower level. This problem is known to be one of the most attractive mathematical problems in the optimization field because of non-convexity of feasible region that it has multiple local optima (Baskan, 2009). Moreover, the driver's behaviours on the network should be taken into account when the traffic signal timings are optimised. When drivers follow the Wardrop's (1952) first principle, the problem is called the "user equilibrium" (UE). On the other hand, it turns to the stochastic user equilibrium (SUE) in the case that the users' face with the decision of route choice between the each Origin-Destination (O-D) pair for a given road network according to perceived travel time. The difference between SUE and UE approaches is that in SUE models each driver is meant to define 'travel costs' individually instead of using a single definition of costs applicable to all drivers. SUE traffic assignment takes into account the variability in driver's perception of cost. This is done by treating the perceived cost on Ant Colony Optimization - Methods and Applications 206 particular path as a random variable distributed across the population of users, and so a different cost for each driver can be modelled. Using the probabilistic choice models, the OD demand is assigned to paths, where the cheapest path attracts the most flow. Road users generally use a variety of routes between their origins and destinations in urban networks based on their perception of travel time. Hence, the SUE is more appropriate than the DUE assignment (Ceylan, 2002).

KEY WORDS

Urban Networks, SUE and UE approaches, DUE assignment

Introduction

A wide range of solution methods to the signal setting problem have been discussed in the literature. Allsop & Charlesworth (1977) found mutually consistent (MC) traffic signal settings and traffic assignment for a medium size road network. In their study, the signal setting and link flows were calculated alternatively by solving the signal setting problem for assumed link flows and carrying out the user equilibrium assignment for the resulting signal settings until convergence was achieved. The obtained mutually consistent signal settings and equilibrium link flows, will, however, in general be non-optimal as has been discussed by Gershwin & Tan (1979) and Dickson

(1981). Abdullaal & LeBlanc (1979) reported the formulation and solution by means of the Hooke-Jeeves' method for an equilibrium network design problem with continuous variables. An unconstrained optimisation problem in which the dependence of equilibrium flows on decision variables was dealt with as one of the decision variables is solved directly by means of the convexcombination method. Suwansirikul et al. (1987) solved equilibrium network design problem that using a direct search based on the Hooke-Jeeves' method for a small test network. The direct search method, however, is computationally intensive, because frequent evaluations of deterministic (or stochastic) user equilibrium traffic assignment are required. Hence, it was emphasized that the proposed method is only suitable for small example road networks. Heydecker & Khoo (1990) proposed a linear constraint approximation to the equilibrium flows with respect to signal setting variables and solved the bi-level problem as a constraint optimisation problem. Using the linear constraint approximation method to solve the bi level problem can be carried out in a number of iterations by which the resulting equilibrium flows are regressed as the signal setting variable changes in a simple linear form. Canteralla et al. (1991) proposed an iterative approach to solve the equilibrium network design problem, in which traffic signal settings are performed in two successive steps; green timing at each junction, and signal co-ordination on the network. Green timing calculation at each junction was based on a mixed binary linear program. Signal coordination for the

whole network was performed by solving a discrete programming model with a total delay minimisation objective function. Yang & Yagar (1995) used derivatives of equilibrium flows and of the corresponding travel times to solve a bi-level program for the equilibrium network design problem for a signal control optimisation. Lee & Machemehl (1998) applied Genetic Algorithm (GA) to individual signalized intersection. Their objective function was a function of green split and the UE links flows. For stage length and cycle time optimization without considering offsets to minimise total travel time, Lee (1998) presented a comparison of GA and simulated annealing with iterative and local search algorithms and showed that different algorithms perform better for different network supply and demand scenarios. Chiou (1999) explored a mixed search procedure to solve an area traffic control optimization problem confined to equilibrium network flows, where good local optima can be effectively found via the gradient projection method. Ceylan & Bell (2004) is proposed GA approach to solve traffic signal control and traffic assignment problem is used to tackle the optimization of signal timings with SUE link flows. The system performance index is defined as the sum Ant Colony Optimization Approach for Optimizing Traffic Signal Timings 207

of a weighted linear combination of delay and number of stops per unit time for all traffic streams. It is found that the GA method is effective and simple to solve equilibrium network design problem.

The optimization methods developed so far to solve signal setting problem are either calculus and mathematically lengthy or are based on the heuristic approaches. Although proposed algorithms are capable of solving signal setting problem for a road network, an efficient algorithm, which is capable of finding the global or near global optima of the upper level signal timing variables considering the equilibrium link flows under SUE conditions, is still needed. This chapter deals with the problem of optimising traffic signal timings without considering offset term taking stochastic user equilibrium (SUE) conditions into account. A bi-level technique and MC approach have been proposed and compared, in which signal setting problem is dealt with as upper level problem whilst the SUE traffic assignment is dealt with as lower level problem. In this chapter, Ant Colony Optimization (ACO) approach which is arisen from the behaviours of real ant colonies is introduced to solve the upper level problem in which traffic signal timings are optimised. Although ACO algorithms are capable of finding global or near global optimum, it may be further improved to locate better global optima for any given problem. In this study, ACO Reduced Search Space (ACORSES) algorithm is used for finding better signal timings. It differs from other approaches in that its feasible search space (FSS) is reduced with best solution obtained so far using the previous information at the each iteration (Baskan et al., 2009). At

the core of ACORSES, ants search randomly the solution within the FSS to reach global optimum by jumping on each direction. At the end of the each iteration, only one ant is near to global optimum. After the first iteration, when global optimum is searched around the best solution of the previous iteration using reduced search space, the ACORSES will reach to the global optimum quickly without being trapped in bad local optimum. In this chapter, signal timings are defined as cycle time and green time for each junction and stage, respectively. The objective function is adopted to minimise the total system cost of network as the system optimum formulation. In order to represent the route choice behaviours of drivers, the probit route choice model is used whilst SUE traffic assignment problem is solved by the method that proposed Sheffi (1985). The effectiveness of the proposed ACORSES algorithm is demonstrated through a numerical experiment. The results showed that the bi-level approach based on ACORSES is considerably effective according to MC approach in terms of the signal timings and the final values of degree of saturation to solve the problem of optimising traffic signal timings under SUE conditions. This chapter is organized as follows. The basic notations are defined in the next section. Section 3 is about the problem formulation. The solution methods for optimising signal timings under SUE conditions are given in Sectio.

Notations

A matrix of signal timings c cycle time for each junction of a given road network $\mathbf{c}(\mathbf{q}, \boldsymbol{\psi}) = [c_a(q_a, \boldsymbol{\psi})]$ the vector of all link travel times, where element $c_a(q_a, \boldsymbol{\psi})$ is travel time on link a as a function of flow on the link itself and the signal setting variables, $\boldsymbol{\psi} = (c, \boldsymbol{\phi})$.

C_a perceived travel time on link a Ant Colony Optimization - c_a measured travel time on link a $h = [h_p ; \forall p \in P_w$

, $\forall w \in W]$ vector of all path flows

I_i intergreen time between signal stages

J number of junctions

K threshold value

L number of links

M numbers of signal stages at a signalised road network

m numbers of signal stages for a signalised junction, $\forall m \in M$

R colony size

s_a degree of saturation on link a

$\mathbf{q} = [q_a ; \forall a \in L]$ vector of flow q_a on link a

$\mathbf{q}^*(\boldsymbol{\psi})$ vector of equilibrium link flows subject to signal parameters

TC total cost of a given road network

$\mathbf{t} = [t_w ; \forall w \in W]$ vector of origin destination flows

$\boldsymbol{\psi}$ signal setting variables

$\boldsymbol{\phi}$ vector of duration of green times

Ω feasible set of signal setting variables

δ link-path incidence matrix

Λ OD-path incidence matrix

$\boldsymbol{\beta}$ vector of search space constraints at ACO

β variance of the perceived travel time

σ a standard error for each link a

$\boldsymbol{\alpha}$ random generated vector as initial solution for ACO

Formulation

The problem of optimising of signal setting variables $\boldsymbol{\psi} = (c, \boldsymbol{\phi})$ without considering offset

term on a road network is defined as bi-level structure. The planners aim to minimise the total cost (TC) of a given road network on the upper level whilst the SUE link flows $\mathbf{q}^*(\psi)$ on the lower level are dealt with altering signal timings. The objective function is therefore to minimise TC with respect to equilibrium link flows $\mathbf{q}^*(\psi)$ subject to signal setting constraints $\psi = (c, \varphi)$. Mathematically the problem is defined as:

$$(\cdot, \cdot^*(\cdot)) (\cdot, \cdot^*(\cdot)) \in \Omega$$

$$\text{Min } TC(\mathbf{q}, \psi)$$

$$\psi \in \Omega$$

$$\psi = \psi$$

$$\in$$

$$\Sigma(1)$$

subject to

$$\min \max$$

$$\min \max$$

$$i \in I$$

cycle time constraints for each junction

(\cdot, \cdot) ; green time constraints for each stage

$$(\cdot)$$

$$m$$

$$i \in I$$

$$c \in C$$

$$\psi \in \Psi$$

$$\varphi \in \Phi$$

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where $\mathbf{q}^*(\psi)$ is implicitly defined by

$$\text{Minimise } (\cdot, \cdot) \mathbf{q}^*(\psi)$$

$$\text{subject to } \mathbf{t} = \Lambda \mathbf{h}, \mathbf{q} = \delta \mathbf{h}, \mathbf{h} \geq 0$$

ACORSES algorithm for optimising of signal timings (upper-level problem)

Ant algorithms were inspired by the observation of real ant colonies. Ants are social insects, that is, insects that live in colonies and whose behaviour is directed more to the survival of the colony as a whole than to that of a single individual component of the colony. Social insects have captured the attention of many scientists because of the high structuration level their colonies can achieve, especially when compared to the relative simplicity of the colony's individuals. An important and interesting behaviour of ant colonies is their foraging behaviour, and, in particular, how ants can find shortest paths between food sources and their nest (Dorigo et al., 1999). Ants are capable of finding the shortest path from food source to their nest or vice versa by smelling pheromones which are chemical substances they leave on the ground while walking. Each ant probabilistically prefers to follow a direction rich in pheromone. This behaviour of real ants can be used to explain how they can find a shortest path (Eshghi & Kasemi, 2006). The ACO is the one of the most recent techniques for approximate optimization methods. The main idea is that it is indirect local communication among the individuals of a population of artificial ants. The core of ant's behavior is the communication between the ants by means of chemical pheromone trails, which enables them to find shortest paths between their nest and food sources. This behaviour of real ant colonies is exploited to solve optimization problems. The general ACO algorithm is illustrated in Fig. 1. The first step consists mainly on the initialization of the pheromone trail. At beginning, each ant builds a complete solution to the problem according to a probabilistic state transition rules. They depend mainly on the state of the pheromone.

Step 1: Initialize Pheromone trail

Step 2: Iteration Repeat for each ant

Solution construction using pheromone trail Update the pheromone trail Until stopping criteria

Fig. 1. A generic ant algorithm Once all ants generate a solution, then global pheromone updating rule is applied in two phases; an evaporation phase, where a fraction of the pheromone evaporates, and a reinforcement phase, where each ant deposits an amount of pheromone which is proportional to the fitness. This process is repeated until stopping criteria is met. In this study, ACORSES algorithm proposed by Baskan et al. (2009) to solve upper-level problem is used to tackle the optimization of signal timings with stochastic equilibrium link flows. The ACORSES algorithm is based on each ant searches only around the best solution of the Ant Colony Optimization previous iteration with reduced search space. It is proposed for improving ACO's solution performance to reach global optimum fairly quickly. The ACORSES is consisted of three main phases; Initialization, pheromone update and solution phase. All of these phases build a complete search to the global optimum. At the beginning of the first iteration, all ants search randomly to the best solution of a given problem within the Feasible Search Space (FSS), and old ant colony is created at initialization phase. After that, quantity of pheromone is updated. In the solution phase, new ant colony is created based on the best solution from the old ant colony. Then, the best solutions of two colonies are compared. At the end of the first iteration, FSS is reduced by β and best solution obtained from the previous iteration is kept. β guides the bounds of search space during the ACORSES application, where β is a vector, β_j ($j=1,2,\dots,n$), and n is the number of variables. The range of the β may be chosen between minimum and maximum bounds of any given problem. Optimum solution is then searched in the reduced search space during the algorithm progress. The ACORSES reaches to the global optimum as ants find their routes in the limited space.

Let the objective function given in Eq. (1) take a set of ψ signal timing variables,

$\psi = (c_1, \phi_1, \dots, c_n, \phi_n)$. On the assumption that each decision variable ψ can take values from a domain $\Omega = [\psi_{min}, \psi_{max}]$ for all $\psi \in \Omega$. The presentation of signal timing variables, $\psi = (c, \phi)$, based on ACO approach is given in Eq. (2).

11 12 13 1 11 12 13 1
 21 22 23 2 21 22 23 2
 1 2 3 1 2 3

where M, J and R are the number of signal stages, the number of junctions at a signalized road network and the value of colony size, respectively. In order to optimise the objective function in ACO real numbers of decision variables are used instead of coding them as in

GA. This is one of the main advantage of ACO approach that it provides to optimise the signal timings at a road network with less mathematically lengthy. Moreover, ACORSES algorithm has ability to reach to the global optimum quickly without being trapped in bad

local optimum because it uses the reduced search space and the values of optimum signal timings are then searched in the reduced search space during the algorithm progress. The ACORSES reaches to the global optimum or near global optimum as ants find their routes in the limited space. In that algorithm, \mathbf{A} consists of the cycle and green timings for each junction and stage at a given road network. In order to provide the constraint of cycle time for each junction, the green timings can be distributed to the all signal stages in a road network as follows (Ceylan & Bell, 2004):

min, min,
 1 1
 1
 ()
 m m
 i
 i i m i k k

$$k k$$

$$i$$

$$k$$

$$c I \phi$$

$$\phi \phi \phi$$

$$\phi = =$$

$$=$$

$$= + -\Sigma -\Sigma$$

$$\Sigma$$

$$i = 1, 2, \dots, m \quad (3)$$

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The solution method for the lower level problem

In this study, probit stochastic user equilibrium (PSUE) model is used to solve lower level problem. Probit model has the advantage of being able to represent perceptual differences in utility of alternatives on a road network. Although it behaves like the logit model for statistically independent paths, it has many advantages in the case of existing correlated paths at a network. For example, two paths that overlap for virtually their whole length are likely to be perceived very similarly by an individual since they have many links in common, but this cannot be captured by a logit model. This can lead to unrealistically high flows being assigned to the common parts of these paths of a given road network. The PSUE model is able to overcome these drawbacks, by supposing path costs are formed from a sum of link costs, with the error distribution (Clark & Watling, 2002). Therefore, PSUE model proposed by Sheffi (1985) is used to find path choice probabilities in order to overcome this drawback. To the best of our knowledge, this is the first time to date that the PSUE model is used to solve SUE traffic assignment problem at the lower level whilst the optimization of signal timings is dealt with upper level problem. Monte-Carlo simulation method is adopted to obtain probit choice probabilities so as to none of the analytical approximation methods can be practically applied to medium or large scale networks due to probit choice probabilities cannot be written in closed form. The advantage of the simulation method is that it does not require the sampling of perceived path travel times only perceived link travel times are sampled at every iteration thus

avoiding path enumeration. The underlying assumption of probit model, the random error term of each alternative is assumed normally distributed. The notation $\xi \sim MVN(\mu, \Sigma)$

indicates that the vector of error terms ξ is multivariate normal (MVN) distributed with mean vector μ and covariance matrix Σ . The algorithm to solve PSUE is given as follows (Sheffi, 1985):

Step 0. Initialization. Set $n=1$

Step 1. Sampling phase. Sample $(n) C_a$ from $C_a \sim N(c_a, \beta c_a)$ for each link a .

Step 2. Based on $(n) C_a$, assign t_w to the shortest path connecting each O-D pair w . Obtain the set of link flows, $(n) q_a$

Step 3. Link flow averaging. Let $(n) (1) (n 1) (n) / q_a n q_a q_a n a = \square - - + \square \square \square \square$

Step 4. Stopping criterion (standard error). , stop. The solution is (n)

q_a . Otherwise, set $n=n+1$ and go to step 1. where K is the predetermined threshold value. In PSUE algorithm, perceived link travel time for each link is random variable that is assumed to be normally distributed with mean equal to the measured link travel time and with variance of related link. According to this

assumption, perceived link travel times for each link are sampled. Then, demand between each O-D pair $w \in W$ assigned to the shortest path to obtain set of link flows, $a \in L$. The link flows have to be averaged at every iteration in order to compute the standard error to test for convergence of algorithm. The PSUE algorithm is terminated when the stopping criterion is met.

Solution methods

The Mutually Consistent (MC) solution for optimising signal timings

The MC solution process was proposed by Allsop (1974) and Gartner (1974). It is an iterative optimisation and assignment procedure that is very similar in nature to other iterative procedures in the literature. In this method, the upper level problem is solved by keeping the flows fixed, and then the traffic assignment problem is solved by keeping the signal timings fixed. MC calculation of signal timings and the corresponding equilibrium flows for bi-level problem starts with an initial assignment and tries to reach a MC solution by solving sequentially an equilibrium assignment problem and a traffic signal setting problem until two successive flow patterns or signal timings are close enough within a specified tolerance. If convergence is reached in a finite number of iterations, the solution is considered to be mutually consistent. This means that the signal settings generate a set of link costs which determine flow pattern such that these settings are optimal for it (Ceylan, 2002). The MC calculation is performed in the following steps.

Step 0. Set $k=0$ for given signal timings $\psi(k)$, find the corresponding equilibrium flows $q^*(\psi(k))$ by solving the problem of PSUE at the lower level.

Step 1. Solve the upper level problem to obtain the optimal signal timings $\psi(k+1)$ for the flows $q^*(\psi(k))$.

Step 2. Update the travel time function of all links according to obtained signal timings $\psi(k+1)$.

Step 3. Calculate the corresponding equilibrium flows $q^*(\psi(k+1))$ by solving the PSUE problem.

At Step 3, method of successive averages (MSA) (Sheffi, 1985) smoothing is applied to the equilibrium link flows in the iterative process in order to overcome fluctuations on equilibrium link flows. The MSA smoothing approach is carried out using the following relationship.

$$q_a^{(k+1)} = (1-\alpha)q_a^{(k)} + \alpha q_a^*(\psi^{(k+1)})$$

where k is the iteration number and a is a set of links in L

Step 4. Solve the upper level problem again to obtain the optimal signal timings $\psi(k+2)$ given by the $q^*(\psi(k+1))$.

Step 5. Compare the values of $\psi(k+2)$ and $\psi(k+1)$, if there is no change between $\psi(k+1)$ and $\psi(k+2)$ then go to Step 6; otherwise, $k=k+1$ and go to Step 2.

Step 6. Stop: $\psi(k+1)$ and $q^*(\psi(k+1))$ are the mutually consistent signal timings and equilibrium flows.

The MC calculation process is terminated when the difference in values of signal timings or of PSUE flows between successive iterations is smaller than a predetermined threshold value.

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The bi-level solution for optimizing signal timings

The problem of optimizing signal timings can be seen as a particular case of the NDP in which the signal timings play a critical role to optimize the performance of the network while the network topological characteristics are fixed. It is also a difficult problem because the evaluation of the upper level objective involves solving the lower level problem for every feasible set of upper level decisions. The complexity of the problem comes from the non-convexity of objective functions and constraints at both levels. In this chapter, the ACORSES algorithm is used to overcome this drawback. Although ACO algorithms are similar to other stochastic optimization techniques, the ACORSES differs from that it uses reduced search space technique to prevent being trapped in bad local optimum (Baskan et al., 2009). The solution steps for the bi-level solution are:

Step 0. *Initialisation*, $k=1$. Set the user-specified ACO parameters (β, R, α); represent the decision variables ψ within the range ψ_{min} and ψ_{max} .

Step 1. If $k=1$ generate the initial random population of signal timings ψ^k as shown in (2).

Else generate random population of signal timings according to best signal timings obtained in Step 8 and β vector of search space constraints. This step plays a critical role because the ACORSES uses the random generated population within the reduced search space at each iteration to prevent being trapped in bad local optimum.

Step 2. Solve the lower level problem by solving the PSUE. This gives an equilibrium link flows for each link a in L .

- Step 3. Calculate the value of objective function using Eq. (1) in order to obtain old ant colony (**Aold**) in which signal timings are presented, for resulting signal timings at Step 1 and the equilibrium link flows resulting in Step 2.
- Step 4. Carry out the pheromone evaporation and updating phases.
- Step 5. Determine the search direction and generate the matrix of length of jump.
- Step 6. According to search direction and generated matrix of length of jump, produce the new ant colony (**Anew**).
- Step 7. Calculate the new values of objective function using produced new ant colony.
- Step 8. Compare the values of objective functions relating to the old and new ant colonies, then we have best decision variables, ψ_{best} .
- Step 9. If the difference between the values of ψ^k and ψ^{k+1} is less than predetermined threshold value, the algorithm is terminated. Else go to Step 1.

5. Numerical example

The test network is chosen that is used by Allsop & Charlesworth (1977) in order to show the performance of the ACORSES to optimise signal setting variables. The network topology and stage configurations are given in Fig. 2a and 2b, where figures are adapted from Ceylan & Bell (2004). Travel demands for each O-D are given in Table 1. This network has 20 O-D pairs and 20 signal setting variables at six signal-controlled junctions. The signal timing constraints are given as follows: c_{min} , $c_{max} = 60, 100$ sec cycle time for each junction $\phi_{min} = 7$ sec minimum green time for signal stages

Conclusions

In this chapter, ACORSES algorithm was used to optimize signal timings on a given road network without considering offset term using bi-level approach and MC solution procedures. The Allsop & Charlesworth's test network was used to denote the performance of the ACORSES in terms of the value of objective function and the degree of saturation on links. According to the results, the final values of degree of saturation from the bi-level solution were not greater than 95% while no links are over-saturated but some of them approach the critical degree of saturation (i.e. 100%) for the MC solution. The ACORSES algorithm is performed on PC Core2 Toshiba machine and each iteration for this test network was not greater than 12.3 sec of CPU time in Visual Basic code. On the other hand, the computation effort for the MC solution on the same machine was carried out for each iteration in less than 15.4 sec of CPU time. The ACORSES algorithm was found considerably successful in terms of the signal timings and the final values of degree of saturation. Although the MC solution gives the similar results in terms of the value of objective function, it produces greater cycle times than the bilevel solution. Moreover, the MC solution was also dependent on the initial set of signal timings and its solution was sensitive to the initial assignment.

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